SGU

Sanjay Ghodawat University, Kolhapur

Established as State Private University under Govt. of Maharashtra. Act No XL, 2017

2018-19
EXM/P/09/01

Year and Program: 2018-2019 M.Sc.	**School of Science**	**Department of Mathematics**
Course Code: MTS 502	**Course Title: Algebra**	**Semester – IV**
Day and Date: Monday 20th May, 2019	**End Semester Examination (ESE)**	**Time:** 3 to 5.30 PM **Max Marks: 100**

Instructions:
1) All questions are compulsory.
2) Figures to the right indicate full marks.
3) Non-programmable calculator is allowed

Q.2		Solve any TWO	Marks	Bloom's Level	CO
	i)	State and prove first theorem of isomorphism.	6	L2	CO1
	ii)	Prove that any two composition series of a group G are isomorphism.	6	L2	CO1
	iii)	Let G be a group and G' be the derived subgroup of a group G. Then show that a) G' is normal subgroup of a group G. b) $\frac{G}{G'}$ is abelian group.	6	L2	CO1
Q.3		**Solve any TWO**			
	i)	Let X be any G-set. Then show that $\|xG\| = (G : G_x)$, where xG is orbit in $x \in X$ under G and $G_x = \{g \in G \mid xg = x\}$.	7	L4	CO2
	ii)	Define p- group. Also show that any finite group G is a p-group if and only if $\| G \|$ is a power of prime p.	7	L4	CO2
	iii)	Let G be a finite group and p divides $\|G\|$, where p is any prime number. If r be the number of Sylow p-subgroups in G then show that $r \equiv 1(\mathrm{mod}\, p)$	7	L2	CO2
Q.4		**Solve any TWO**			
	i)	Let R be a ring and $f(x), g(x)$ be non-zero polynomials in $R[x]$, where $\deg f(x) = n$ and $\deg g(x) = m$. If $f(x) + g(x)$ and $f(x).g(x)$ are non-zero polynomials in $R[x]$, then show that a) $\deg[f(x) + g(x)] \leq \max\{m, n\}$ b) $\deg[f(x) \cdot g(x)] \leq m + n$.	7	L4	CO3
	ii)	Show that the polynomial $f(x) = x^4+4$ can be factorized into linear factors in $Z_5[x]$, Where $Z_5[x]$ is ring of polynomial over integer modulo 5.	7	L5	CO3

ESE

	iii)	Show that the following polynomials is irreducible over Q, Where Q is a field of rational number. a) $f(x) = 8x^3 - 6x - 1$ b) $f(x) = x^4 + x^3 + x^2 + x + 1$	7	L3	CO3
Q.5		**Solve any FOUR**			
	i)	Prove that every principle ideal ring R satisfy ascending chain condition for ideals.	5	L5	CO4
	ii)	Show that ring R is Noetherian ring if and only if every ideal of R is finitely generated.	5	L2	CO4
	iii)	If R is Noetherian ring then show that maximum condition for ideals hold in ring R.	5	L2	CO4
	iv)	If maximum condition for ideals hold in ring R then show that every ideal in R is finitely generated.	5	L2	CO4
	v)	Show that any quotient ring of a Noetherian ring is Noetherian.	5	L2	CO4
	vi)	If a ring R is Artinian ring then show that any homomorphic image of R is also Artinian ring.	5	L2	CO4
Q.6		**Solve any FOUR**			
	i)	Let M be any R–module prove that a) $0 \cdot m = 0$ for all $m \in M$ b) $r \cdot 0 = 0$ for all $r \in R$ c) $(-r) \cdot m = (-rm) = r \cdot (-m)$ for all $m \in M, r \in R$.	5	L5	CO5
	ii)	If N_1 and N_2 are submodules of R –module M then show that $(N_1 + N_2)$ is a submodule of M.	5	L2	CO5
	iii)	Let M and N be modules and let $f: M \to N$ be R – homomorphism. Then show that f is one-one if and only if $\ker f = \{0\}$.	5	L5	CO5
	iv)	Prove that any homomorphic image of an R- module M is isomorphic with its suitable quotient module.	5	L5	CO5
	v)	Let A and B be submodules of a R – module M. Prove that $\frac{A+B}{A} \cong \frac{B}{A \cap B}$.	5	L5	CO5
	vi)	Let M be a simple R –module. Then show that any non-zero homomorphism defined on M is an isomorphism.	5	L2	CO5

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Year and Program: 2018-2019 **M.Sc. II**

School of Science

Department of Mathematics

Course Code – MTS 502
Day and Date – Monday 20th May, 2019
PRN number –

Course Title –Algebra
End Semester Examination
Seat no-

Semester – IV
Time: 2:30 to 3:00 pm
Max Marks: 100
Answer Booklet No.-

Students' Signature -

Invigilator's Signature –

Instructions:

1) All questions are compulsory.
2) **Attempt Q.1 within first 30 minutes.**
3) Each MCQ type question is followed by four plausible alternatives, Tick (√) the correct one.
4) Answer to question 1 should be written in the question paper and submit to the Jr. Supervisor.
5) If you tick more than one option it will not be evaluated
6) Figures to the right indicate full marks
7) Use **Blue ball pen** only.

		Marks	Bloom's Level	CO
Q.1	Tick Mark correct alternative			
i)	Which of the following statement is not true? a) Every subgroup of a solvable group is solvable. b) Homomorphic image of a solvable group is solvable. c) Any quotient group of a solvable group is solvable. d) Quotient group $\frac{G}{N}$ is a solvable group then group G is solvable.	2	L2	CO1
ii)	Let G be a finite group of order 33. Which of the following option is not true? a) G contains a normal subgroup of order 11. b) G is simple. c) G is cyclic. d) G is abelian.	2	L3	CO2
iii)	Which of the following is zero of $f(x) = x^5 + 3x^3 + x^2 + 2x$ in $Z_5[x]$, where $Z_5[x]$ is a polynomial ring over integer modulo 5. a) 1 b) 2 c) 3 d) 4	2	L3	CO3

ESE

iv)	A ring of integer is______ a) Noetherian ring. b) Artinian ring. c) satisfy descending chain condition. d) do not satisfy ascending chain condition.	2	L4	CO4
v)	A ring R is called Noetherian ring, if_______ a) R satisfy ascending chain condition for ideals. b) R satisfy descending chain condition for ideals. c) minimal condition for ideals hold in R. d) some ideals of R is not finitely generated.	2	L2	CO4
vi)	Which of the following is Noetherian ring as well as Artinian ring? a) A ring of integer. b) Principle ideal domain. c) Every field. d) A ring $Z[i]$, where $Z[i] = \{a + bi \mid a, b \epsilon Z\}$, Z is integer and $i = \sqrt{-1}$.	2	L3	CO4
vii)	Which of the following statement is not true? a) Sum of two submodules of R-module M is a submodule of R-module M. b) Intersection of two submodules of R-module M is a submodule of R-module M. c) Union of any chain of submodules of a given R- module M is a R-submodule of M. d) Every subset of an R-module M is a submodule of M.	2	L2	CO4
viii)	Which of the following is the identity of a quotient module $\frac{M}{N}$? a) $\{0\}$ b) $\{1\}$ c) N d) M	2	L1	CO5
ix)	A R-module M is called simple if it has_____ submodule/submodules. a) 1 b) 2 c) 3 d) 4	2	L1	CO5
x)	Let M, N be R-modules and $f: M \to N$ be R homomorphism. Then which of the following statement is not true? a) Kernel of f is a submodule of module M. b) Kernel of f is a submodule of module N. c) Image of f is a submodule of module N. d) Image of f is itself a R-module.	2	L2	CO5

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