Sanjay Ghodawat University, Kolhapur
Established as State Private University under Govt. of Maharashtra.
Act No XL, 2017

2018-19
EXM/P/09/00

Year and Program:2018-19, M. Sc.	**School of Science**	**Department of Mathematics**
Course Code –MTS 510	**Course Title –Partial Differential Equations**	**Semester – II**
Day and Date – Friday 31/05/2019	**End Semester Examination**	**Time:** 30 min. 2·30 to 3·00 pm **Max Marks: 100**
PRN number –	**Seat no-**	**Answer Booklet No.-**
Students' Signature -	A	**Invigilator's Signature –**

Instructions:

1) All questions are compulsory.
2) **Attempt Q.1 within first 30 minutes.**
3) Each MCQ type question is followed by four plausible alternatives, Tick (√) the correct one.
4) Answer to question 1 should be written in the question paper and submit to the Jr. Supervisor.
5) If you tick more than one option it will not be evaluated
6) Figures to the right indicate full marks
7) Use **Blue ball pen** only.

Q.1	Tick Mark correct alternative	Marks	Bloom's Level	Cos
1	The partial differential equation which represents all surfaces of the form $z = xy + f\left(x^2 + y^2\right)$, is given by $(a)\, x^2 + y^2 + px + qy = 0,$ $(b)\, px + qy - x^2 - y^2 = 0,$ $(c)\, yp - xq + x^2 - y^2 = 0,$ $(d)\, yp + xq + x^2 - y^2 = 0.$	02	L_3	CO1
2	The condition that the partial differential equations $f(x,y,z,p,q) = 0,\ g(x,y,z,p,q) = 0$ are compatible if	02	L_3	CO2

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(a) Every solution of $f(x,y,z,p,q)=0$ is also a solution of $g(x,y,z,p,q)=0$,

(b) Every solution of $g(x,y,z,p,q)=0$ is also a solution of $f(x,y,z,p,q)=0$,

(c) The equations $f(x,y,z,p,q)=0$ and $g(x,y,z,p,q)=0$ have common solution,

(d) There exists many common solutions of $f(x,y,z,p,q)=0$ and $g(x,y,z,p,q)=0$

3	The integral surface of the partial differential equation $yp-xq=0$ through $x^2=z, y=0$ is $(a)\, z=x^2+y^2$, $(b)\, z^2=x^2+y^2$, $(c)\, z=x+y$, $(d)\, z=\sqrt{x^2+y^2}$.	02	L_2	CO3
4	The partial differential equation $u_{xx}+xyu_{xy}=0$ is an ellipse if $(a)\, x\neq 0,\ y>0$, $(b)\, x<0,\ y<0$, $(c)\, x>0,\ y<0$, $(d)\, x<0, y>0$.	02	L_3	CO4
5	The characteristic curves of the partial differential equation $x^2u_{xx}-y^2u_{yy}=0$ are $(a)\, x+y=c_1,\ x-y=c_2$, $(b)\, x^2+y^2=c_1,\ x^2-y^2=c_2$, $(c)\, (x+y)^2=c_1,\ (x-y)^2=c_2$, $(d)\, xy=c_1,\ \frac{y}{x}=c_2$.	02	L_3	CO4
6	The vertical displacement $u(x,t)$ of an infinitely long elastic string is governed by the initial value problem $u_{tt}=4u_{xx},\ -\infty<x<\infty,\ t>0,$ $u(x,0)=-x,\ u_t(x,0)=0.$ Then the value of $u(2,2)$ is equal to	02	L_4	CO4

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$(a) 2,$ $(b) -2,$
$(c) 4,$ $(d) -4.$

7	The partial differential equation $u_{xx} + xu_{yy} = 0,\ x > 0$ is (a) parabolic, (b) hyperbolic, (c) elliptical, (d) circular.	02	L_2	CO4
8	The problem of determining a function $u(x, y)$ which is harmonic inside a finite region D and f is a continuous function on the boundary B of D and $u = f$ on B, is called (a) interior Dirichlet problem, (b) exterior Dirichlet problem, (c) interior Neumann problem, (d) exterior Neumann problem.	02	L_2	CO5
9	If a harmonic function vanishes everywhere on the boundary B of a finite region D then (a) f is identically zero in D, (b) f is identically zero in $\overline{D} = D \cup B$, (c) f is constant in D, (d) f is constant in $\overline{D} = D \cup B$.	02	L_2	CO5
10	One dimensional wave equation which describes the vibrations of an elastic string is characterised by the partial differential equation $(a)\ y_x = \frac{1}{c^2} y_{tt},$ $(b)\ y_t = \frac{1}{c^2} y_{xx},$ $(c)\ y_{xx} = \frac{1}{c^2} y_{tt},$ $(d)\ y_{xx} + y_{yy} = 0.$	02	L_2	CO5

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Year and Program: 2018-19, M. Sc.	**School of Science**	**Department of Mathematics**
Course Code: MTS 510	**Course Title: Partial Differential Equation**	**Semester – IV**
Day and Date: Friday 31/05/2019	**End Semester Examination (ESE)**	**Time:** 2.5 Hrs. 3.00 to 5.30 pm **Max Marks: 100**

Instructions:
1) All questions are compulsory.
2) Figures to the right indicate full marks.
3) Non-programmable calculator is allowed

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Q.N			Marks	Bloom's Level	CO
Q.2	a)	Prove that $F(u,v)=0$ is the general solution of the Lagrange's differential equation $P(x,y,z)p+Q(x,y,z)q=R(x,y,z)$, where F is arbitrary and $u(x,y,z)=c_1$, $v(x,y,z)=c_2$ are two independent solutions of the equation $\frac{dx}{P}=\frac{dy}{Q}=\frac{dz}{R}$.	6	L_3	CO1
		OR			
	a)	Let $\bar{X}=(P,Q,R)$ be a vector such that $\bar{X}\cdot curl\bar{X}=0$ and μ is an arbitrary differentiable function of x,y,z, then prove that $(\mu\bar{X})\cdot curl(\mu\bar{X})=0$.	6	L_3	CO1
	b)	Show that the partial differential equations $xp-yq=0$ and $z(xp+yq)=2xy$ are compatible and find a one parameter family of common solution.	6	L_3	CO1
Q.3	a)	Describe Charpits method of solving a first order partial differential equation $f(x,y,z,p,q)=0$.	7	L_4	CO2
		OR			
	a)	Find the complete integral of the partial differential equation $z(p^2+q^2)+px+qy=0$ by Charpit's method.	7	L_4	CO2
	b)	Solve the partial differential equation $z^2u_x^2u_y^2u_z^2+u_x^2u_y^2-u_z^2=0$.	7	L_4	CO2

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Q.4	a)	What is Cauchy problem? Discuss how a general solution of linear partial differential equation may be used to determine the integral surface, which passes through a given curve.	7	L_4	CO3
		OR			
	b)	Prove that there exists a solution $z(x,y)$ of the partial differential equation $P(x,y,z)p+Q(x,y,z)q=R(x,y,z)$ defined in some neighborhood of the initial curve $\Gamma_0: x=x_0(s), y=y_0(s)$, which satisfies the initial condition $z(x_o(s),y_o(s))=z_o(s)$, and $\frac{dy_0}{ds}P(x_0(s),y_0(s),z_0(s))-\frac{dx_0}{ds}Q(x_0(s),y_0(s),z_0(s))\neq 0$, for $a\leq s\leq b$,	7	L_4	CO3
	c)	Find the integral surface of the partial differential equation $x(z+z)p+(xz+2yz+2y)q=z(z+1)$ passing through the curve $x_o=s, y_o=0, z_0=2s$.	7	L_4	CO3
Q.5	a)	By a suitable change of the independent variables x,y to any other independent variables ξ,η, reduce the equation $Rr+Ss+Tt+g(x,y,u,u_x,u_y)=0$ to parabolic form $u_{\eta\eta}=\phi(\xi,\eta,u,u_\xi,u_\eta)$ when $S^2-4RT=0$.	10	L_5	CO4
		OR			
	b)	Obtain d'Alembert's solution of the one dimensional wave equation which describes the vibrations of a semi- infinite string.	10	L_5	CO4
	c)	A string is stretched and fastened to two points l apart. Motion is started by displacing the string in the form $y(x,0)=y_0\sin^3\left(\frac{\pi x}{l}\right)$ from which it is released at a time $t=0$. Show that their displacement of any point at a distance x from one end at time t is given by $y(x,t)=\frac{y_0}{4}\left[3\sin\left(\frac{\pi x}{l}\right)cox\left(\frac{\pi ct}{l}\right)-\sin\left(\frac{3\pi x}{l}\right)cox\left(\frac{3\pi ct}{l}\right)\right]$.	10	L_5	CO4

Q.6	a)	Show by using the method of separable variables that the general solution of the Laplace's equation in spherical polar coordinates is $u(r,\theta,\phi)=\sum_{n=0}^{\infty}\left(\alpha_n r^n+\beta_n\frac{1}{r^{n+1}}\right)S_n(\theta,\phi)$, where $S_n(\theta,\phi)=\sum_{m=0}^{n}P_n^m(\mu)(A_{nm}\cos m\phi+B_{nm}\sin m\phi)$, and $\mu=\cot\theta$, $P_n^m(\mu)$ is the Legendre function and $\alpha_n,\beta_n,A_{nm},B_{nm}$ are constants.	12	L_5	CO5
		OR			
	a)	Solve the equation $\nabla^2 u=0,\ \ 0\le\theta\le 2\pi,\ r\ge a,$ $u(a,\theta)=f(\theta),\ r=a,$ where $f(\theta)$ is a continuous function of θ on the boundary $r=a$.	12	L_5	CO5
	b)	Let D be a region bounded by a simple closed, piecewise smooth curve B. If $u(x,y)$ is harmonic in D and continuous in $\bar{D}=D\cup B$. Then show that $u(x,y)$ attains its maximum and minimum value on the boundary B of D .	08	L_4	CO5

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