05
05
05
05
05
05
05
6

Sanjay Ghodawat University, Kolhapur
Established as State Private University under Govt. of Maharashtra.
Act No XL, 2017

2018-19
EXM/P/09/00

Year and Program: **2018-19: M.Sc.-I**	**School of Science**	**Department of Mathematics**
Course Code – MTS 504	**Course Title – Complex Analysis**	**Semester – II**
Day and Date – Wednesday 22 May 2019	**End Semester Examination**	**Time:** 2:30 to 3:00 pm
		Max Marks: 100
PRN number –	**Seat no-**	**Answer Booklet No.-**
Students' Signature -		**Invigilator's Signature –**

Instructions:

1) All questions are compulsory.
2) **Attempt Q.1 within first 30 minutes.**
3) Each MCQ type question is followed by four plausible alternatives, Tick (√) the correct one.
4) Answer to question 1 should be written in the question paper and submit to the Jr. Supervisor.
5) If you tick more than one option it will not be evaluated.
6) Figures to the right indicate full marks.
7) Use **Blue ball pen** only.

Q.1	Tick Mark correct alternative	Marks	Bloom's Level	Cos
1)	If a Mobius transformation S has more than two fixed points, then S must be A) Unit function B) Constant C)Identity D) Zero	01	L_1	CO1
2)	The equation $\alpha(x^2+y^2)+\beta x+\gamma y+\delta=0$ $(\alpha,\beta,\gamma,\delta\in R)$ represents a circle if A) $\alpha=0$, $\delta=0$ C) $\alpha=0$, $\beta\neq0$ B) $\alpha=0$ D) $\alpha\neq0$	01	L_2	CO1
3)	A non constant entire function cannot be bounded. This is statement of A) Moreras theorem C) Lebnitzs theorem B) Taylors theorem D) Liouville's theorem	01	L_1	CO2

page 1

FSE

4) Let C be a circle $|z|=\frac{3}{2}$ in the complex plane that is oriented in the counter clockwise direction. The value of a for which 01 L_4 CO2

$$\int_C \left(\frac{z+1}{z^2-3z+2}-\frac{a}{z-1}\right)dz=0 \text{ is}$$

A) 1 B) 2 C) -1 D) -2

5) If γ is closed rectifiable curve in C, then for $a \notin \{\gamma\}$, the index of γ with respect to point a is 01 L_1 CO3

A) $n(\gamma;a)=\frac{1}{2\pi}\int_\gamma (z-a)^{-1}dz$ C) $n(\gamma;a)=\frac{1}{2\pi i}\int_\gamma (z-a)dz$

B) $n(\gamma;a)=\frac{1}{2\pi i}\int_\gamma (z-a)^{-1}dz$ D) $n(\gamma;a)=\frac{1}{2}\int_\gamma \frac{1}{(z-a)}dz$

6) The value of $\int_C \frac{(3z^2+7z+1)}{z+1}dz$ where C is circle $|z|=\frac{1}{2}$ is 01 L_3 CO3

A) $2\pi i$ B) πi C) $\frac{\pi i}{2}$ D) 0

7) The value of the integral $\int_{|z|=2} \frac{2z+1}{z^2+z+1}dz$ is 01 L2 CO4

A) 0 B) $2\pi i$ C) $4\pi i$ D) πi

8) If $0<|z-1|<2$, the expansion of $f(z)=\frac{z}{(z-1)(z-2)}$ is 01 L_4 CO4

A) $f(z)=\sum_{n=0}^{\infty}\frac{2^n-1}{2^n}z^{n-1}$

B) $f(z)=\frac{1}{2z}-\sum_{n=0}^{\infty}\frac{1}{z^{n+1}}-\frac{1}{4}\sum_{n=0}^{\infty}\left(\frac{z}{2}\right)^n$

C) $f(z)=\frac{1}{2z}-\sum_{n=0}^{\infty}\frac{1}{z^{n+1}}+\frac{1}{4}\sum_{n=0}^{\infty}\left(\frac{z}{2}\right)^n$

D) $f(z)=-\frac{1}{2(z-1)}-\frac{3}{4}\sum_{n=0}^{\infty}\left(\frac{z-1}{2}\right)^n$

page 2

ESE

9)	If $z=a$ is an isolated singularity of f and $\sum_{z=-\infty}^{\infty} a_n(z-a)^n$ is its Laurent expansion in ann(a;0,R), then, $z=a$ is an essential singularity, if A) $a_n \neq 0$ for infinitely many negative n B) $a_n \neq 0$ for all integer n D) $a_n \neq 0$ for many positive n D) $a_n = 0$ for all integer n	01	L_3	CO4
10)	In the series expansion of $f(z)=\frac{1}{z-1}-\frac{1}{z-2}$ valid in the region $\lvert z\rvert > 2$, then the coefficient of $\frac{1}{z^2}$ is A) 1 C) -2 B) 2 D) -1	01	L_4	CO4
11)	The poles of the function $f(z)=\frac{\sin z}{\cos z}$ are at A) $n\pi$ C) $\frac{n\pi}{2}$ B) $\frac{2n\pi}{3}$, n is integer D) $\frac{(2n+1)\pi}{2}$, n is integer	01	L_3	CO4
12)	$f(z)=\frac{\sin z}{(z-\pi)^3}$ has the pole of order A) 1 C) 3 B) 2 D) 4	01	L_2	CO4
13)	If f has a pole of order m at $z=a$ and $g(z)=(z-a)^m f(z)$, then A) $res(f;a)=\frac{1}{(m)!}g^{(m)}(a)$ C) $res(f;a)=\frac{1}{(m-1)!}g(a)$ B) $res(f;a)=\frac{1}{(m-1)!}g^{(m-1)}(a)$ D) $res(f;a)=g^{(m-1)}(a)$	01	L_1	CO4

14) The value of the integral $\int_{|z|=1} \frac{e^z - 2}{e^z - 2z - 1} dz$ is | 01 | L4 | CO5

A) $2\pi i$ B) 1 C) 0 D) πi

15) Let G be a region, $\{f_n\} \subseteq H(G)$. If $f_n \to f$ in $H(G)$ and each f_n never vanishes on G, then which one of the following statement is true | 01 | L1 | CO5

A) $f_n.f \equiv 2$
C) $f_n.f \equiv 0$
B) $f_n.f \equiv 1$
D) $f \equiv 0$ or f never vanishes

16) Which one of the following is not an example of simply connected region | 01 | L2 | CO5

A) ann(0; 6,5)
C) ann(0;1,2)
B) ann(0;0,2)
D) ann(0;1,4)

17) Let f and g be analytic in a nbhd of $\overline{B}(a;R)$ with no zeros on $\gamma : |z-a| = R$. If $|g(z)| < |f(z)|$ on γ, then | 01 | L1 | CO5

A) $f+g$ and $f-g$ have same number of zeros inside γ

B) f and g have same number of zeros inside γ

C) f and $f+g$ have same number of zeros inside γ

D) g and $f+g$ have same number of zeros inside γ

18) The roots of the equation $z^3 - 6z + 1 = 0$ lie between the circles | 01 | L_4 | CO5

A) $|z| = 1$ and $|z| = 3$
C) $|z| = 3$ and $|z| = 4$
B) $|z| = 0$ and $|z| = 2$
D) $|z| = 0$ and $|z| = 1$

19) If f & G are analytic in neighborhood of $\overline{B}(a;R)$ then | 01 | L_1 | CO5

A) $Z_f = Z_g$ B) $Z_f < Z_g$ C) $Z_f > Z_g$ D) $Z_f \neq Z_g$ always

20) Which one of the following functions is not meromorphic function | 01 | L_2 | CO5

A) $f(z) = \frac{1}{z-2}$
C) $f(z) = \frac{1}{\sin z}$
B) $f(z) = \frac{1}{z-6}$
D) $f(z) = e^{1/z}$

page 4
ESE

Year and Program: 2019; M.Sc.-I	**School of Science**	**Department of Mathematics**
Course Code: MTS504	**Course Title: Complex Analysis**	**Semester – IV**
Day and Date: Wednesday, 22nd May 2019	**End Semester Examination (ESE)**	**Time:** 3:00 pm to 5:00 pm
		Max Marks: 100

Instructions:
1) All questions are compulsory.
2) Figures to the right indicate full marks.
3) Non-programmable calculator is allowed.

Q.N			Marks	Bloom's Level	Cos
Q.2		Attempt any **TWO**			
	a)	Prove that the necessary condition for a function $f(z)=u+iv$ to be analytic at all the points in a region G are $\frac{\partial u}{\partial x}=\frac{\partial v}{\partial y}$ & $\frac{\partial u}{\partial y}=-\frac{\partial v}{\partial x}$, provided $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$ exists.	06	L_2	CO1
	b)	If S is Mobious transformation then prove that S is composition of translation, dilation and the inverse.	06	L_3	CO1
	c)	Find the radius convergence of $f(z)=\sum_{n=0}^{\infty}\frac{n!}{n^n}z^n$	06	L_2	CO1
Q.3	a)	Let $\phi:[a,b]\times[c,d]\to C$ be a continuous function and $g:[c,d]\to C$ defined by by $g(t)=\int_a^b \phi(s,t)ds$. Prove that (i) g is continuous. (ii) if $\frac{\partial \phi}{\partial t}$ exists and is a continuous function on $[a,b]\times[c,d]$ then g is continuously differentiable and $g'(t)=\int_a^b \frac{\partial \phi}{\partial t} ds$	10	L_4	CO2
		OR			
	a)	Let f be analytic in $B(a,r)$. Show that $f(z)=\sum_{n=0}^{\infty}a_n(z-a)^n$ for $\|z-a\|<r$ where $a_n=\frac{1}{n!}f^{(n)}(a)$ and this series has radius of	10	L_4	CO2

page 1

ESE

		convergence $\geq$ R			
	b)	Evaluate $\int_{\gamma} \frac{e^{iz}}{z^2} dz;\ \gamma(t) = e^{it}\ (0 \leq t \leq 2\pi)$	04	L1	CO2
Q.4		Attempt any **TWO**			
	a)	If f is analytics in a region G and a is a point in G with $\lvert f(a) \rvert \geq \lvert f(z) \rvert$ for all z in G, then prove that f must be constant function.	07	L_2	CO3
	b)	Let $\gamma : [0,1] \rightarrow C$ be a closed rectifiable curve and $a \notin \{\gamma\}$. Prove that $\frac{1}{2\pi i} \int_{\gamma} \frac{dz}{z-a}$ is an integer.	07	L_3	CO3
	c)	Solve $\int_{\lvert z \rvert = 2} \frac{z - 3\cos(z)}{\left(z - \frac{\pi}{2}\right)^2} dz$	07	L_4	CO3
Q.5		Attempt any **TWO**			
	a)	State and prove Goursat's theorem	10	L_5	CO4
	b)	Let $z = a$ be an isolated singularity of a function f. Then $z = a$ is removable singularity of f iff $\lim_{z \to a} (z-a) f(z) = 0$	10	L_2	CO4
	c)	Calculate residue of i) $\frac{z^2}{(z-1)(z-2)^2}$ ii) $\frac{z+1}{z^2 - 2z}$	10	L_1	CO4
Q.6	a)	State and prove Schwarz's lemma	10	L_3	CO5
		OR			
	a)	State and prove Rouches's theorem	10	L_3	CO5
	b)	Attempt any **TWO**			
		i) Show that the equation $e^{z-\alpha} = z^n$, where n is positive integer $\alpha > 1$, has exactly n roots in $B(0,1)$ counted according to their multiplicities.	5	L_5	CO5
		ii) Evaluate $\int_{\lvert z \rvert = 1} \frac{e^z - 2}{z^3 - 6z + 8} dz$ by using Rouche's theorem	5	L_5	CO5
		iii) Find $\int_{\lvert z \rvert = 2} \frac{f'(z)}{f(z)} dz$, where $f(z) = \frac{(z-1)^3 (z-4)^4}{(z+i)^2 (z-i)(z+1-i)^3}$	5	L_3	CO5

*****

page 2

ESE